Radiative transfer effects in natural convection above fires

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This paper describes the results of examining the influence of radiative heat transfer on turbulent natural convection above fires in an atmosphere of constant potential temperature, under both the 'opaque' and 'transparent' approximations. It turns out that on the basis of the over-all approximations introduced in this investigation, the former case reduces to that of no radiative transfer. For the latter case, the initial fire size, the energy release rate (initial temperature difference) and the absorption coefficient have been regarded as independent parameters. The solution curves presented cover a range over which these parameters are expected to vary in practice.

1. Introduction

For an exact theoretical investigation of the role played by the radiative mode of heat transfer in the dynamics of a rising column of hot gases above a fire, one has to solve the three conservation equations of fluid mechanics coupled with the integro-differential equation of radiative transfer. The formulation of the first part of the problem, namely the fundamental equations, will depend upon whether one is dealing with a laminar or a turbulent flow. Experiments on the burning of dishes of liquid fuel have shown that the flow field is turbulent at a height greater than about one or two times the diameter of the dish. Owing to this, as well as to the fact that the radiation effect will be significant only in larger fires which are turbulent right from the start, it seems more realistic to confine the present discussion of radiation effects to turbulent fields of flow. Having set up the system of equations for such a flow, before any further progress can be made one needs to know the composition of the plume at any height and the absorption coefficient of each component at the temperature and pressure prevailing at that level. The former property can be obtained by solving the problem of mixing of the products of combustion with the constituents of the atmosphere outside, and one could perhaps do this for the turbulent case, at least to the same order of accuracy as one determines the other variables in the problem. As regards the latter property, from a theoretical stand-point one must know the spectral distribution, line shape, etc., for each component of the plume. This knowledge does not seem to exist for the products of combustion that may be formed above fires at the temperatures prevailing there.

One can, however, make measurements on emissivities from which the absorption coefficient k^* can be calculated. These have been made by Hottel &

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Mangelsdorf (1935) for carbon dioxide and water vapour up to quite high temperatures. This data could be usefully employed for certain 'clean' fires. For fires which might contain other constituents, one needs additional measurements. It seems appropriate, from the point of view of a theoretical analysis, to work out the solution for an expected range of the absorption coefficient, being guided in this estimation by the above data as also by other data on flame emissivities. This paper is an account of such an attempt. No change in composition of the plume with height—either due to change of temperature or mixing—is taken into account. In other words, it is assumed that the products of a combustion in the plume above a fire consist of a 'single' component whose radiation properties—emission and absorption—remain constant with height. Further, we make use of the asymptotic solutions of the integro-differential equation of radiative heat transfer for both the opaque and transparent approximations. Finally, we assume that the emissivity and absorptivity are the same, the outside atmosphere being calm, with constant potential temperature and emitting black-body radiation.

2. The fundamental equations and numerical solution

For a cylindrically-symmetric convection column, let $\bar{x}$ be the axial co-ordinate measured vertically upward and let r be the radial co-ordinate. The corresponding velocity components u, v, the density ρ, the pressure p, and all other fluid properties, are assumed to be local mean values. Thus turbulent components are averaged out or included in other terms as shear stress τ or heat flux q. If the vertical pressure distribution is given by the hydrostatic approximation, we have

$$p = p_0 - \gamma_\infty \bar{x}, \tag{1}$$

where p_0 is the standard pressure and γ_∞ the specific weight of the fluid outside the plume at infinity. The equations of conservation of mass, momentum and energy are then

$$\frac{\partial}{\partial \bar{x}}(\gamma r u) + \frac{\partial}{\partial r}(\gamma r v) = 0, \tag{2}$$

$$r u \frac{\partial u}{\partial \bar{x}} + r v \frac{\partial u}{\partial r} = \frac{r(\gamma_\infty - \gamma)}{\rho} + \frac{1}{\rho}\frac{\partial}{\partial r}(r\tau), \tag{3}$$

$$r\left(u\frac{\partial h}{\partial \bar{x}} + v\frac{\partial h}{\partial r} + ug\right) = -\frac{1}{\rho}\frac{\partial}{\partial r}(rq) + rH. \tag{4}$$

Here γ is the specific weight of the fluid inside the plume, h its enthalpy per unit volume, g the gravitational acceleration, and τ and q the vertical shear stress and radial flux, being given in turbulent flow by $\overline{\rho u'v'}$ and $\overline{c_p \rho u'T'}$, respectively, where u', v', and T' are the fluctuating components of u, v, and T, the absolute temperature. H is the heating per unit volume due to radiation flux. In equation (4) the dissipation function and vertical heat flux have been neglected. The above system of equations have to be supplemented by the equation of radiative heat transfer (Kourganoff 1953), namely

$$dI/d\mathbf{s} = k^*\{B - I(\mathbf{s})\}, \tag{5}$$

where I is the intensity of radiation at a point integrated over all frequencies, k^* the 'grey' absorption coefficient for the fluid, B the Planck's function, and $\mathbf{s}$ an element of length. The heating rate H per unit volume due to radiation is given by the integral over all solid angles ω of $dI/d\mathbf{s}$, namely

$$H = -\int \frac{dI}{d\mathbf{s}} d\omega = -4\pi k^* B + k^* \int I(\mathbf{s})\, d\omega. \tag{6}$$

(i) *Transparent approximation*

In equation (6) the first term gives the amount of radiation emitted by an element, while the second term gives that absorbed by it from its surroundings, which include other elements in the plume as well as the boundaries. Since the mean free path of the radiation is $(k^*)^{-1}$, it is clear that if $(k^*)^{-1}$ is much greater than some characteristic length, say the plume width, then the element under consideration will be emitting radiation to and receiving it from the surroundings only. This is the so-called 'transparent' approximation. Inserting the value of Planck's function B, and assuming that the radiation emitted by the surroundings is black-body radiation at the absolute temperature T_∞ and that k^* is the same inside and outside the plume, we have

$$H = -4\pi k^* \sigma (T^4 - T_\infty^4), \tag{7}$$

where σ is the Stefan constant.

(ii) *Opaque approximation*

The other limiting case which suggests itself occurs when $(k^*)^{-1}$ is much smaller than some characteristic length in the problem. It is known that, for such a case, the radiation has diffusive properties acting like conduction with conductivity proportional to T^3. Mathematically one could obtain this by solving equation (5) in the usual way, expanding B as a Taylor series in the optical depth (defined as the integral of k^* over any path) and substituting in equation (6) (see, for example, Chandrasekhar 1939). One retains only three terms and neglects the higher ones in the expansion (for they can be shown to be much smaller for this approximation). One obtains, corresponding to equation (7) for the other approximation, the equation

$$H = -\frac{4\pi}{3k^*} \nabla^2 B = -\frac{4\pi}{3k^*} \left\{ \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial}{\partial r} \right) + \frac{\partial^2}{\partial x^2} \right\} B. \tag{8}$$

After putting the two values of H given by (7) and (8) in the energy equation above, we proceed as in the previous work by Murgai & Emmons (1960); namely, we substitute potential temperature and density in the above equations, integrate them with respect to r from 0 to ∞, and define the mean values of velocity $\bar{u}$, plume-width $\bar{b}$, and specific gravity difference $-\overline{\Delta\gamma_0}/\gamma_0$ as in equations (9) to (12) of the previous paper mentioned above. A subscript zero now indicates potential temperature (T_0) and specific weight (γ_0). In terms of these mean values, our equations for an atmosphere with constant potential temperature, for the transparent approximation, become

$$\frac{d}{d\bar{x}}(\mathfrak{b}^2\bar{u}) = \alpha\mathfrak{b}^2\bar{u}, \tag{9}$$

$$\frac{d}{d\bar{x}}(\mathfrak{b}^2\bar{u}^2) = g\mathfrak{b}^2\frac{\overline{\Delta\gamma_0}}{\gamma_0}, \tag{10}$$

$$\frac{d}{d\bar{x}}\left(\mathfrak{b}^2\bar{u}\frac{\overline{\Delta\gamma_0}}{\gamma_0}\right) = -\frac{4k^*\sigma p_0^{(3\kappa-4)/\kappa}\mathfrak{b}_0^{\frac{1}{2}}T_{\infty 0}^3}{I^*\rho_0 c_p}\mathfrak{b}^2\left(\frac{\rho_{\infty 0}^4}{\rho_0^4}-1\right), \tag{11}$$

κ being the exponent of the adiabatic law, p_0 the ratio of the pressure to some standard pressure, c_p the specific heat at constant pressure, $\mathfrak{b}_0$ the initial value of $\mathfrak{b}$, ρ_0 the potential density inside the plume, $\rho_{\infty 0}$ the potential density of the outside atmosphere, and I^* in equation (11) the shape factor denoted by I in relation (12) of Murgai & Emmons (1960).

For the opaque case one should neglect the vertical diffusion term $\partial^2 B/\partial\bar{x}^2$, in order to be consistent with the rest of the formulation, before substituting the value of H in equation (4), and then one should proceed to integrate it with respect to r. Thus one gets, on the right-hand side of the energy equation, the term

$$\left(\frac{16\sigma r T_0^3 p_0^{(3\kappa-4)/\kappa}}{3k^*\rho_0 c_p}\frac{\partial T_0}{\partial r}\right)_{r=\infty}. \tag{11a}$$

For the steady-state problem of gravitational convection above fires, in an infinite atmosphere, the most natural boundary condition at the edges of the plume seems to be

$$T_0 = T_{\infty 0}, \quad \text{or} \quad (r\,\partial T_0/\partial r)_{r=\infty} = 0.$$

This leads to the result that, within the framework of these approximations (boundary-layer equations, infinite régime, etc.), the opaque case reduces to the case of no radiative transfer.†

We now define the following dimensionless variables:

$$\left.\begin{aligned} &x = \frac{\alpha\bar{x}}{\mathfrak{b}_0}, \quad b = \frac{\mathfrak{b}}{\mathfrak{b}_0}, \quad u = \left\{\frac{\alpha}{(\overline{\Delta\gamma_0/\gamma_0})_0}\right\}^{\frac{1}{2}}\frac{\bar{u}}{(\mathfrak{b}_0 g)^{\frac{1}{2}}},\\ &\lambda = \frac{(\overline{\Delta\gamma_0/\gamma_0})}{(\overline{\Delta\gamma_0/\gamma_0})_0}, \quad \delta = \left(\frac{T_0 - T_{\infty 0}}{T_{\infty 0}}\right)_0 = \left(\frac{T-T_\infty}{T}\right)_0,\\ &\phi = \frac{4k^*\sigma T_{\infty 0}^3 p_0^{(3\kappa-4)/\kappa}\mathfrak{b}_0^{\frac{1}{2}}}{\rho_0 c_p g^{\frac{1}{2}}\alpha^{\frac{1}{2}}I^*(\overline{\Delta\gamma_0/\gamma_0})_0^{\frac{3}{2}}}. \end{aligned}\right\} \tag{12}$$

(Thus ϕ is the dimensionless 'radiation number'.)

This leads to the final form of the conservation equations for the transparent case

$$(d/dx)(b^2u) = bu, \tag{13}$$

$$(d/dx)(b^2u^2) = b^2\lambda, \tag{14}$$

$$(d/dx)(b^2u\lambda) = -b^2\phi\{(1+\lambda\delta)^4-1\}. \tag{15}$$

† The expression given by 11(*a*) above will not be zero, for example, for the case of an axisymmetric fire surrounded by another ring of fire, there then being a finite amount of radiative transfer at the boundaries.

The boundary conditions are

$$b = 1, \quad \lambda = 1, \quad u = u_0 = 1 \text{ (say)}, \quad \text{for} \quad x = 0. \tag{16}$$

The case $\phi = 0$ also represents, in a certain sense only, the opaque case.

Generally speaking, u_0 should be a parameter, but it is expected that, in a problem primarily meant to estimate the effects of radiation on the dynamics of a plume, the variation of its initial velocity will have no significant effect. For this reason we have worked out the numerical solutions of the above equations with δ and ϕ as parameters, varying from 1 to 5 and 0 to 0·5, respectively.

In order to do this, we made the following substitutions: $bu = v$, $b^2u = w$, $b^2\lambda u = f$.

The equations (13) to (15) and the boundary conditions (16) then become

$$dw/dx = v, \tag{17}$$

$$dv^4/dx = 2fw, \tag{18}$$

$$\frac{df}{dx} = -\frac{w^2}{v^2}\phi\left\{\left(1+\delta\frac{f}{w}\right)^4 - 1\right\}, \tag{19}$$

and

$$v = w = f = 1, \quad \text{for} \quad x = 0. \tag{20}$$

Starting with $x = 0$, the values of v, w and f at a small Δx were found by straightforward integration after giving a certain preassigned value to them in this interval. This was repeated for this Δx till the successive calculations converged. The values thus found provided a new set of starting conditions for the next interval. This process was carried out till f was $O(0)$, after which the equations had the following closed solution, namely $w - w_0 = v_0(x - x_0)$, v_0, w_0 being the values of v and w respectively at $x = x_0$, the value of x at which $f = 0$. The details of numerical analysis are available on request. Figures 1, 2 and 3 are for b, u and λ for $\delta = 2{\cdot}0$ and ϕ values given on the curves.

It will be apparent from the curves that while the system loses buoyancy in a very small fraction of its height of ascent it has sufficient momentum to go up. It is quite instructive, for the purpose of discussion, to define a 50 % height for buoyancy and momentum as the height during which, in the process of ascent of the plume, they are reduced to half of their initial value. We have determined these heights for $k^* = 0{\cdot}01\ \text{cm}^{-1}$ for various values of δ and ϕ.† These are given in table 1. It is apparent that in almost all cases the 50 % height for buoyancy is reached when the rising plume has hardly lost any momentum. The fire size b_0 for the same values of δ and ϕ is proportional to $(k^*)^{-2}$, and for any other value of the absorption coefficient, b_0 and the corresponding maximum heights reached can be found from those given in the table by appropriate scaling. For an optically thick plume as distinguished from an optically thin one, one would expect on physical grounds that it would not lose buoyancy as soon as the latter does (because of a 'diffusive contact' with the outside atmosphere), and it would therefore 'last' longer. This fact is borne out even by the above approximate solution of the

† k^* is defined in terms of the emmissivity ϵ as $\operatorname{Lt}_{x\to 0} \dfrac{d\epsilon}{dx}$, where x is the path length. k^* calculated in this manner from Hottel's data for carbon dioxide is $0{\cdot}1\ \text{cm}^{-1}$.

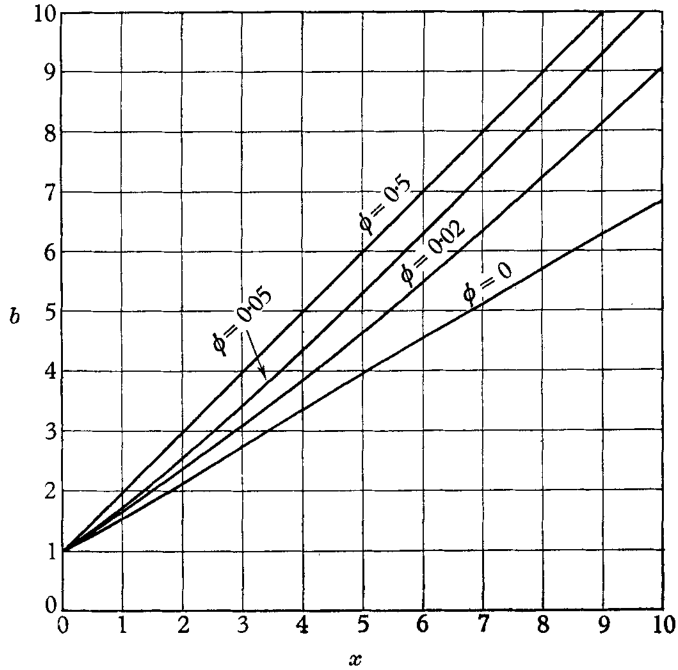

FIGURE 1. The variation of plume width b with height x and radiation number. Parameter δ is the ratio of the initial difference of temperature of the plume and the outside atmosphere and the temperature of the latter. $\delta = 2$.

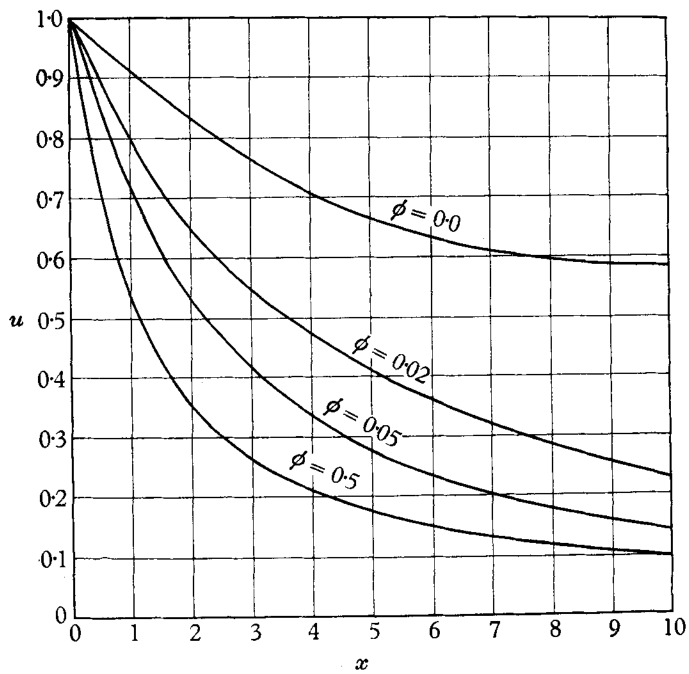

FIGURE 2. The variation of vertical velocity u with height x and the radiation number. Parameter δ is the ratio of the initial difference of temperature of the plume and the outside atmosphere and the temperature of the latter. $\delta = 2$.

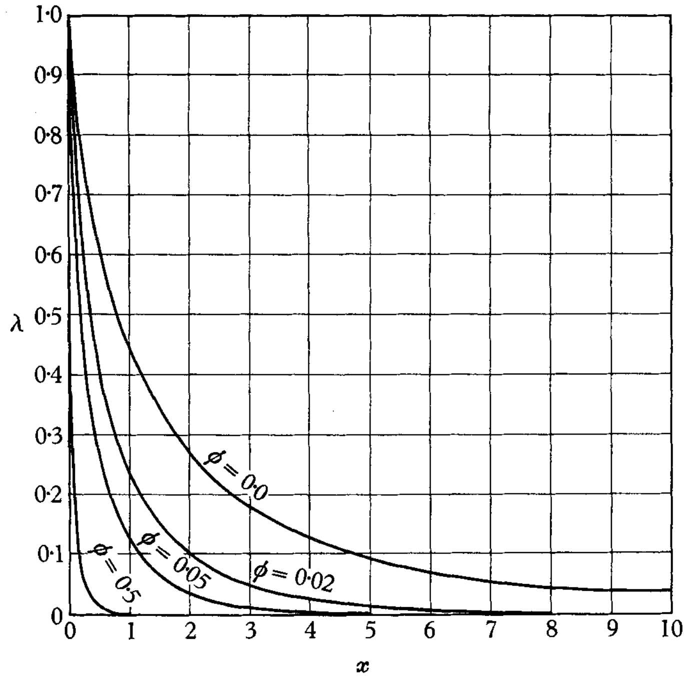

FIGURE 3. The variation of buoyancy λ with height x and the radiation number. Parameter δ is the ratio of the initial difference of temperature of the plume and the outside atmosphere and the temperature of the latter. $\delta = 2$.

δ	ϕ	T^* °C	$\bar{b}_0 \times 10^{-5}$ km	X_1	X_2	$\bar{h}_1 \times 10^{-5}$ km	$\bar{h}_2 \times 10^{-5}$ km
1·0	0·0200	327	0·0242	6·375	0·6375	1·540	0·1540
	0·2000	—	2·415	1·975	0·2125	47·69	5·131
	0·5000	—	15·09	1·390	0·0875	209·8	13·21
2·0	0·0200	627	0·1884	3·600	0·3500	0·6781	0·6592
	0·0500	—	1·177	2·250	0·2000	26·49	2·354
	0·5000	—	17·7	1·150	0·0210	1354	24·72
3·0	0·0020	927	0·0065	7·875	0·6000	0·5134	0·0391
	0·0200	—	0·6520	2·430	0·1900	15·84	1·239
	0·0500	—	4·075	1·650	0·0900	67·24	3·667
	0·5000	—	407·5	1·050	0·0075	4279	30·56
4·0	0·0020	1227	0·0155	5·8500	0·4600	0·9041	0·0713
	0·0200	—	1·545	1·850	0·1000	28·58	0·3090
	0·0500	—	9·462	1·380	0·0500	133·1	4·821
5·0	0·0020	1527	0·0302	4·470	0·3250	1·349	0·0981
	0·0500	—	18·87	1·250	0·0400	235·8	7·546
	0·5000	—	1887	1·010	0·0020	2105 × 10	37·73

TABLE 1. Fifty per cent. heights $\bar{h}_1$ and $\bar{h}_2$ for velocity and for buoyancy respectively for typical fire sizes $\bar{b}_0$ and their initial temperatures T^* for $k^* = 0{\cdot}01\,\mathrm{cm}^{-1}$ corresponding to some values of the dimensionless parameters ϕ and δ. The heights $\bar{h}_1$ and $\bar{h}_2$ and the initial fire size $\bar{b}_0$ are in kilometres, while the temperature T^* is in degrees Centigrade. X_1 and X_2 are the dimensionless heights corresponding to $\bar{h}_1$ and $\bar{h}_2$, respectively

opaque case,† which might, at best, be regarded as a particular solution for this approximation and not the only one.

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† The referee commented that in other dynamical problems the maximum effect usually occurs in the opaque approximation. The author thinks that the answer to this question can, perhaps, be found in looking at the complete solution rather than the asymptotic cases as hitherto done. This attempt is being made.